Journal of Heat and Mass Transfer, Vol. 9, No. 9, Sept. 1966, pp. 987–992.

[20] Nonnemacher, T. F., "Jacobi Polynomial Solution of the Boltzmann Equation and Some Remarks about the Connection between the Spherical Harmonics and Case Methods," *Developments in Transport Theory*, edited by E. Inonu and P. F. Zweifel, Academic Press, New York, 1967, pp. 343–356.

[21] Giovanelli, R. G., "Radiative Transfer in Non-Uniform Media," *Australian Journal of Physics*, Vol. 12, No. 2, June 1959, pp. 164–170.

[22] Traugott, S. C., "Radiative Heat-Flux Potential for a Nongrey Gas," *AIAA Journal*, Vol. 4, No. 3, March 1966, pp. 541–542.

[23] Shen, S. F., "On the Differential Equation Approximation in Radiative Gas Dynamics," Rept. NOL TR 67-23, May 1967, Naval Ordnance Lab.

[24] Heaslet, M. A. and Warming, R. F., "Application of Invariance Principles to a Radiative Transfer Problem in a Homogeneous Spherical Medium," *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 5, No. 5, Sept.–Oct. 1965, pp. 669–682.

[25] Ambarzumian, V. A., "A Point-Source of Light within a Scattering Medium," Bulletin 6, 1945, Erevan Astronomical Observatory.

[26] Heaslet, M. A. and Warming, R. F., "Radiative Transport and Wall Temperature Slip in an Absorbing Planar Medium," *International Journal of Heat and Mass Transfer*, Vol. 8, No. 6, June 1965, pp. 979–994.

[27] Viskanta, R. and Crosbie, A. L., "Radiative Transfer through a Spherical Shell of an Absorbing-Emitting Gray Medium," *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 7, No. 6, Nov.–Dec. 1967, pp. 871–889.

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Formulation of Two-Dimensional Radiant Heat Flux for Absorbing-Emitting Plane Layer with Nonisothermal Bounding Walls

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A two-dimensional approximate formulation for the radiant heat flux and its divergence is developed for the case of an absorbing-emitting medium bounded by nonisothermal parallel plates. For the optically thin limit, the present analysis approaches the rigorous exact solution. The formulation is believed to be simple enough to be useful for a variety of practical problems of radiative equilibrium and combined conduction-convection and radiation pertaining to the geometry and boundary conditions of this model. The applicability of this formulation and the "two-dimensionality" effects are demonstrated for the case of radiative equilibrium in the presence of a gray gas of constant properties.

Nomenclature

B = radiosity
C = general constant
C_p = specific heat at constant pressure
e = blackbody emissive power
E_n = exponential integral $E_n(\tau) = \int_0^1 t^{n-2} \exp\left(-\frac{\tau}{t}\right)dt$
$F(\tau,r)$ = function defined by Eq. (7)
$G(\tau,r)$ = function defined by Eq. (10)
I = intensity of radiation
q = heat-flux rate
r = the ratio X/Y
r_0 = the ratio $X/(Y_0 - Y)$
s = coordinate measured in the direction of a pencil of rays
t = dummy variable of integration
T = absolute temperature
x = optical distance in X direction $\int_0^x \kappa dX$
X = Cartesian coordinate
Y = Cartesian coordinate
θ = polar angle measured from normal
κ = absorption coefficient
μ = $\cos\theta$
σ = Stefan-Boltzmann constant
τ = optical distance in the Y direction, $\int_0^y \kappa dY$
ϕ = polar angle measured from the positive X direction
ω = solid angle

Subscripts and superscripts

0 = displacement between the walls, also used for the intensity emitted from a solid surface
1 = lower plate at $X < 0$
2 = upper plate at $X < 0$
e = upper wall at $X > 0$
r = radiation
s = in the direction of a pencil of rays
w = lower wall at $X > 0$
ν = specular
$(\ )^+$ = in the positive Y direction
$(\ )^-$ = in the negative Y direction

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Introduction

The first step in solving a heat-transfer problem is to formulate the conservation of energy equation. Insofar as convection and conduction are considered, the heat flux is controlled by the local temperature and the temperature gradient, leading to a differential formulation that is solvable for proper boundary conditions.

However, in many physical situations, especially at high temperatures, one must consider also the radiant heat flux. When the medium is absorbing-emitting the formulation of the radiant heat flux enters into the energy equation, thereby introducing a serious difficulty for a solution. Contrary to conduction and convection, the radiant heat transfer is, generally, a long range phenomena. Since each volume element in the medium can absorb, emit, and exchange heat with every element of the medium and with any solid surface in the surroundings, the radiant heat flux at each point is given by a complicated integral that involves integration with respect to optical distance, direction, and wavelength. It is clearly seen that a general solution to an energy equation that contains such a complicated integral formulation is very difficult.

Because of this difficulty most of the problems in the recent past treated only a one-dimensional case where the temperature is a function of only one coordinate.[1,2] Under this category we can find, for example, solutions for the case of radiative equilibrium of an absorbing-emitting gas between two parallel plates,[3] interaction between conduction and radiation[4,5] and conduction, convection and radiation.[6,7]

In fact, the one-dimensional formula derived for the case of a plane layer is also often used as an approximation for cases where the temperature field is essentially two-dimensional. Thus, very frequently the one-dimensional formula is used in treating boundary-layer problems with pure convection and radiation (Ref. 8, p. 275) and combined convection, conduction, and radiation (Ref. 8, pp. 284 and Ref. 9).

The first multidimensional formulation is the Rosseland diffusion approximation which is valid at large optical depths but fails near solid boundaries. In spite of its limitation, it has been used for examples in the study of downstream effects in a boundary-layer flow.[10,11] Modification of the thick approximation was performed by Deissler[12] introducing the temperature jump at the wall for pure radiation thereby accounting for the failure of the thick approximation at the boundaries. A method based on a directional averaging of the intensity and a moment method are given in Refs. 13 and 14, respectively. Both are subject to the question of whether the formulation is indeed a valid approximation, since the functional form of the intensity with respect to direction is arbitrarily assumed and the main feature of radiant heat transfer as an "action-at-a-distance" phenomenon closely tied to the bounding walls condition is not satisfied. This shortcoming was partially overcome by the use of the Modified Differential Approximation in which the special effect of the external radiation sources is taken into account.[15]

An alternative way to treat a multidimensional problem is to rely heavily on numerical means. As an example, one can find first some noncoupled radiant heat-transfer calculations done by numerical integration.[16,17] Recently DeSoto[18] extended the numerical procedure to get a solution to the problem of coupled radiation, conduction, and convection in entrance region flow by an iterative method.

The present paper attempts to suggest a meaningful step towards attacking a two-dimensional case with a solely integral formulation. The main object is to arrive at a simple tractable expression for the divergence of the radiant heat flux by employing an approximation to the gas self-absorption such that the boundary conditions remain in their rigorous integral form. Such formulation is rigorously correct in the optically thin limit and hopefully should hold fairly well (depending on other conditions) up to intermediate optical thicknesses. The particular case treated here (Fig. 1) is basically still a plane geometry. However, since the walls are not isothermal, the temperature field is two-dimensional. Use of the expression for the divergence of radiant flux in the conservation of energy equation may yield solutions to many practical problems pertaining to this model. This may include problems of radiative equilibrium, combined conduction and radiation, Couette flows with jump boundary conditions, flows in channels and finally boundary-layer flows over solid surfaces.

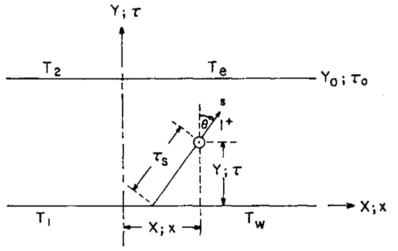

Fig. 1 Physical model and coordinate system.

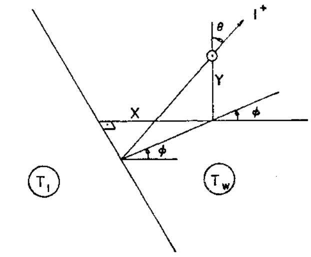

Fig. 2 Geometrical system.

Analysis

The equation of radiative transfer under local thermodynamic equilibrium for an absorbing-emitting, nonscattering diffuse medium with an index of refraction of unity is

$$dI/ds = -\kappa I + \kappa e/\pi \tag{1}$$

The analysis is carried out for a gray gas; however, this is done only for simplifying the equations and the results presented are equally valid on a spectral basis.

Equation (1) may be formally integrated along a path of a pencil of rays to yield

$$I = I_0 \exp(-\tau_s) + \frac{1}{\pi}\int_0^{\tau_s} e(t_s)\exp[-(\tau_s - t_s)]dt_s \tag{2}$$

The divergence of the radiant flux is controlled by the balance between the energy emitted per unit volume and the radiant energy absorbed from the impinging intensities from all directions, and is equal to

$$\frac{\Delta q_r}{\kappa} = 4e - \int_{4\pi} I d\omega = 4e - \int_{4\pi} I_0 e^{-\tau_s} d\omega - \frac{1}{\pi}\int_{4\pi} d\omega \int_0^{\tau_s} e(t_s)\exp[-(\tau_s - t_s)]dt_s \tag{3}$$

In this paper, we propose to obtain an approximate formulation by approximating the $e(t_s)$ value in Eq. (3). The first consequence is that at the optically thin limit case the solution will approach the rigorous exact solution. As the optical thickness increases, the validity of this approximation will depend on the validity of our approximation for $e(t_s)$.

Considering now the particular geometry shown in Fig. 1, it is convenient to break the integrals in Eq. (3) into two integrals involving the upper half-sphere and the lower one. For the lower half-sphere, we can write

$$\int_{2\pi} I^+ d\omega = \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi/2} I^+(0)\exp(-\tau_s)\sin\theta d\theta + \frac{1}{\pi}\int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi/2} \sin\theta d\theta \int_0^{\tau_s} e(t_s)\exp[-(\tau_s - t_s)]dt_s \tag{4}$$

with a similar expression for the integration over the upper-half sphere.

Figure 2 is a three-dimensional sketch of the present geometry. Using the geometrical relations of this figure, one can determine for every point (X,Y) and direction characterized by θ and ϕ whether $I^+(0)$, the intensity from the

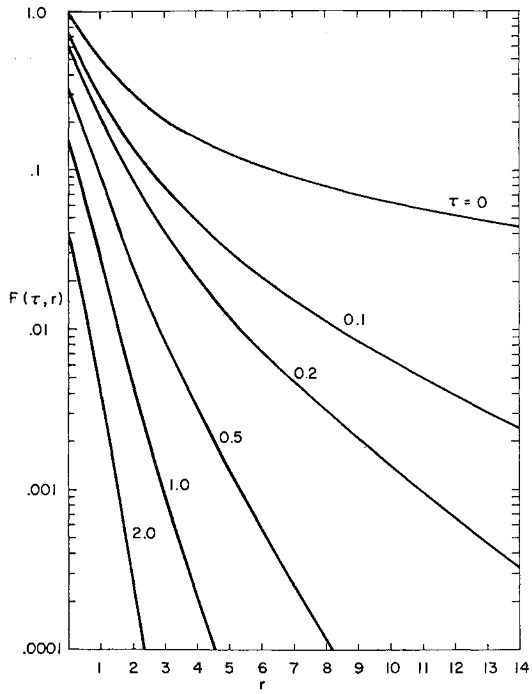

Fig. 3 The function $F(\tau,r)$.

lower plate, is either I_w or I_1. The result is

$$I^+(0) = \begin{cases} I_w \begin{cases} \frac{\pi}{2} \leq \phi \leq \frac{3\pi}{2} & 0 \leq \theta < \frac{\pi}{2} \\ -\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2} & 0 \leq \theta < \tan^{-1}\left(\frac{X}{Y\cos\phi}\right) \end{cases} \\ I_1 \left\{ -\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2} \right. & \tan^{-1}\left(\frac{X}{Y\cos\phi}\right) < \theta < \frac{\pi}{2} \end{cases} \tag{5}$$

At this point, we invoke the following approximations. The absorption coefficient κ is assumed to be constant. This results in the relation $\tau_s = \tau/\mu$.

Next, we approximate the emissive power along the line of integration from 0 to τ_s [where τ_s is at a position (x,τ)] to be $e(x,t)$ instead of $e(t_s)$ (t is the dummy variable for $\tau = t_s\mu$). This simplification is justified as follows: for any "left" line of integration in a direction (θ,ϕ), we have a counter "right" line of integration in a direction $[\theta,\phi + \pi]$. For many practical applications, the variation of the temperature profile in the x direction (at any τ location) is monotonic. Thus, by taking $e(t)$ instead of $e(t_s)$ for both left and right lines of integration we get, in effect, a higher value for one line and a lower value for the other one. After integrating with respect to direction, the errors would partially cancel each other. At this point, it should be mentioned that if the emissive power distribution is linear with respect to x the aforementioned approximation is rigorously correct. Finally, notice that this approximation is applied to the gaseous self-absorption only. When the optical distance between the two plates is small the relative contribution of the integral terms containing $e(t_s)$ is small compared to the contribution of the gas emission and the wall intensities. Thus, the effect of this approximation on the final result is of second order only. This, of course, also means that the solution is rigorously correct in the case of the optically thin limit as stated earlier.

In addition, we assume the walls to be diffuse and thus, I_w, I_1, I_e, I_2 do not depend on the polar angle. In this case, the radiosities from the wall are given by $B_w = \pi I_w$, $B_1 = \pi I_1, \ldots$, etc.

Substituting $I^+(0)$ from Eq. (5) into Eq. (4), taking advantage of the aforementioned simplifications, and considering the analog of Eqs. (4) and (5) for I^-, the final result for the divergence of the heat flux is

$$\frac{\Delta q_r}{\kappa} = 4e(X,Y) - 2B_wE_2(\tau) + (B_w - B_1)F(\tau,r) - 2B_eE_2(\tau_0 - \tau) + (B_e - B_2)F(\tau_0 - \tau,r_0) - 2\int_0^{\tau_0} e(X,t)E_1(|\tau - t|)dt \tag{6}$$

where

$$r = X/Y, \quad r_0 = X/(Y_0 - Y)$$

and

$$F(\tau,r) = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} d\phi \int_0^{\cos[\tan^{-1}(r/\cos\phi)]} \exp\left(-\frac{\tau}{\mu}\right) d\mu \tag{7}$$

For the limiting cases when r is 0 or ∞, the value of this function is

$$F(\tau,0) = E_2(\tau),\ F(\tau,\infty) = 0$$

The complete behavior of the function $F(\tau,r)$ which was obtained numerically using the Gaussian integration procedure, is given in Fig. 3.

Inspection of the preceding equations shows that when $X/Y \to \infty$, $F(\tau,\infty) \to 0$ and the formulation approaches the rigorous one-dimensional exact formulation. Also, for the limit when $B_1 \simeq B_w$ and $B_2 \simeq B_e$ the formulation approaches the exact one. This gives us some confidence as to the validity of this approximation and also indicates under what conditions the approximation is valid.

Heat Transfer

In this problem, we will be interested, generally, in the net heat flux in the Y direction. This can be calculated through the relation

$$q_r = \int_{4\pi} I \cos\theta d\theta = \int_{\phi=0}^{2\pi} d\phi \int_0^1 I^+\mu d\mu - \int_{\phi=0}^{2\pi} d\phi \int_0^1 I^-\mu d\mu \tag{8}$$

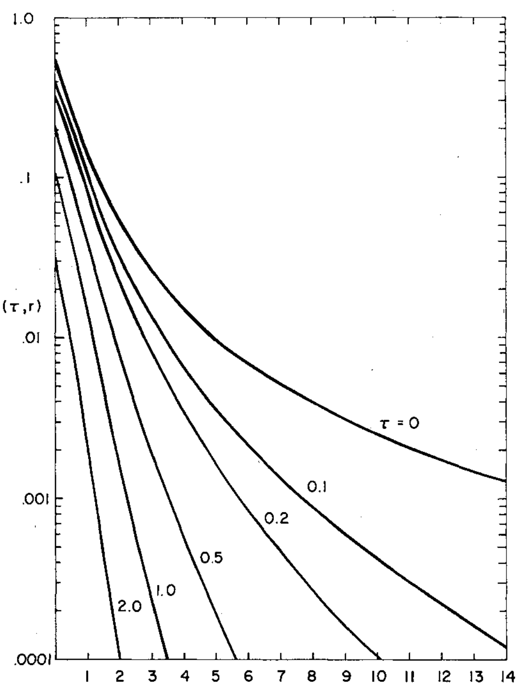

Fig. 4 The function $G(\tau,r)$.

Using the same procedures as before, the final result for the net heat flux in the Y direction is

$$q_r = 2B_wE_3(\tau) - (B_w - B_1)G(\tau,r) - 2B_eE_3(\tau_0 - \tau) + (B_e - B_2)G(\tau_0 - \tau,r_0) + 2\int_0^\tau e(X,t)E_2(\tau - t)dt - 2\int_\tau^{\tau_0} e(X,t)E_2(t - \tau)dt \quad (9)$$

where

$$G(\tau,r) = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} d\phi \int_0^{\cos[\tan^{-1}(r/\cos\phi)]} \mu \exp\left(-\frac{\tau}{\mu}\right) d\mu \quad (10)$$

$G(\tau,r)$, also obtained by numerical integration, is given in Fig. 4. The limiting cases could be observed also directly from Eq. (10) to yield

$$G(\tau,0) = E_3(\tau),\ G(\tau,\infty) = 0$$

Again, one can see that Eq. (9) is reduced to the exact one-dimensional case either when $B_w = B_1$ and $B_e = B_2$ or when $X/Y \to \infty$.

As mentioned earlier, the results of Eqs. (6) and (9) are valid for the nongray case when the spectral quantities replace the total ones; namely, when κ, τ, B_w, B_e, B_1, B_2, e are replaced by κ_ν, τ_ν, $B_{w\nu}$, $B_{e\nu}$, $B_{1\nu}$, $B_{2\nu}$, and e_ν, respectively, followed by an integration with respect to wavelength.

Though the results presented here are developed for the case of temperature independent absorption (or spectral absorption) coefficient, it is felt that as an additional approximation the present formulation can handle also a variable absorption coefficient analysis. Physically this means that we approximate the absorption coefficient along a line of integration from 0 to τ_s to be the same as along 0 to τ (for constant X). This approximation is good when the variation of the absorption coefficient in the X direction is a slowly varying function. Also, we note that an integration along any left line accompanied by an integration along the counter right line decreases the possible error.

Optically Thin Limit

Under optically thin conditions, namely $\tau_0 \ll 1$, when terms of the $0(\tau_0)$ are deleted, Eqs. (6–10) take the simple form

$$\Delta q_r/\kappa = 4e(X,Y) - 2B_w + (B_w - B_1)F(0,r) - 2B_e + (B_e - B_2)F(0,r_0) \quad (11)$$

$$q_r = B_w - (B_w - B_1)G(0,r) - B_e + (B_e - B_2)G(0,r_0) \quad (12)$$

where

$$F(0,r) = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \cos\left[\tan^{-1}\left(\frac{r}{\cos\phi}\right)\right] d\phi \quad (13)$$

and

$$G(0,r) = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \cos^2\left[\tan^{-1}\left(\frac{r}{\cos\phi}\right)\right] d\phi \quad (14)$$

Notice that for the thin limit the formulations (11–14) are rigorously accurate.

For $r > 1$, very simple and extremely good asymptotic expansions for these functions are found. The results are

$$F(0,r) = (2/\pi)[1/r - 1/3r^3 + \tfrac{1}{5}1/r^5 - \ldots]\ r > 1 \quad (15)$$

$$G(0,r) = 1/4r^2 - 3/16r^4 + 5/32r^6 - \ldots\ r > 1 \quad (16)$$

Comparison with the exact solution shows that when three terms of the series are used the results are very good even for $r = 1.5$. The error at $r = 1.5$ for F is about 1% and less than 5% for G. For $r > 4$, one term is sufficient both for F and G, with error less than 2%.

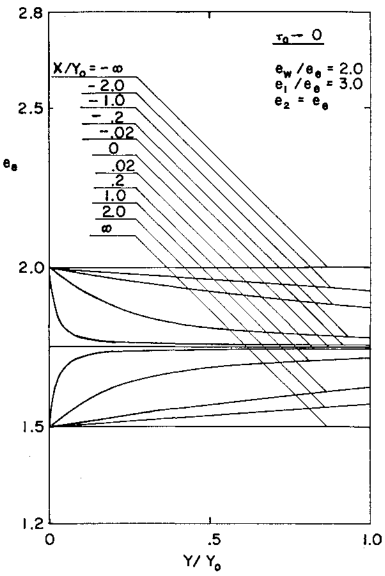

Fig. 5 Emissive power profile for $\tau_0 = 0.0$.

For $r < 1$, F and G can be approximated in the region 0 to 1 with a straight line on Figs. 3 and 4, assuming

$$F = \exp(-C_1r)\quad C_1 = 0.693 \quad (17)$$

$$G = 0.5\exp(-C_2r)\quad C_2 = 1.228 \quad (18)$$

C_1 and C_2 were determined from the exact numerical evaluation of this integral at $r = 1$, indicating this approximation is best near 0 and 1. Obviously the choice of 1 is arbitrary and one may choose other points between zero and one in order to approximate this function in this region. In the absence of an exact evaluation, even the series expansions (15) and (16) may be useful in determining C_1 and C_2.

Though an exact numerical evaluation of the integrals (13) and (14) and even (7) and (10) presents no real problem, the derivation of the approximations (15–18) illustrates that the present formulation may be useful even when a computer is not available.

Example—Radiative Equilibrium Problem

Consider the geometry described in Fig. 1 for the case of black walls, gray gas, and constant absorption coefficient.

A numerical solution of Eq. (6) obtained by an iterative procedure was carried out for the optical thickness $\tau_0 = 0.1$ and 1.0 and for $e_e = e_2$, $e_w/e_e = 2.0$ and $e_1/e_e = 3.0$. For the optically thin limit, Eq. (11) yields

$$e(X,Y) = (e_w + e_e)/2 - [(e_w - e_1)/4]F(0,r) - [(e_e - e_2)/4]F(0,r_0)\quad X > 0 \quad (19)$$

For $X < 0$, we reverse the roles of e_w with e_1 and e_e with e_2 and the result is

$$e(X,Y) = (e_1 + e_2)/2 - [(e_1 - e_w)/4]F(0,-r) - [(e_2 - e_e)/4]F(0,-r_0)\quad X < 0 \quad (20)$$

The values of the function F are given in Fig. 3 or alternatively one may consult the approximate Eqs. (15) and (17).

The results of the dimensionless emissive power profile are reported in Figs. 5, 6, and 7 for $\tau = 0$, 0.1, and 1.0, respectively. As expected for large $|X|$, the solution approaches the one-dimensional result for two isothermal plates. The sequence of the emissive power profiles obtained when X decreases displays the following interesting features. First, one may observe the possible appearance of a maximum in the emissive power profile for $X > 0$. This maximum is a characteristic of the action-at-a-distance of radiative transfer

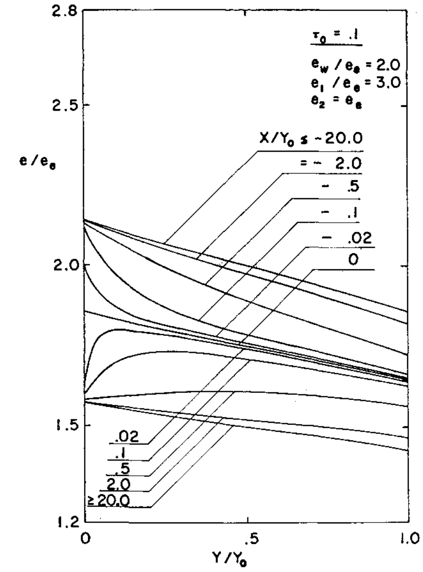

Fig. 6 Emissive power profile for $\tau_0 = 0.1$.

and cannot be obtained by any localized or diffusion approximation. This maximum occurs because as X decreases a volume element of gas sees the hotter wall (e_1) and absorbs direct radiation from this wall. In the optically thin limit, the gas offers no attenuation to the incoming radiation from plate 1, and therefore the maximum temperature appears near the upper walls e at the point where the solid angle that views the left wall 1 is maximum. As the optical thickness increases, attenuation decreases the intensity received from plate 1 for an element far away from this plate and the maximum emissive power will appear at a point where the combination of maximum view angle and minimum attenuation result in the maximum intensity absorbed from plate 1. Because of this, one can observe that the point of maximum temperature moves in Figs. 5, 6, and 7 to the left for increasing τ_0 as well as decreasing X. Another interesting point is the appearance of a discontinuity in the temperature at the point $x = 0$, $\tau = 0$. (A similar discontinuity will appear at $x = 0$, $\tau = \tau_0$ for $e_e \neq e_2$.) Approaching $x = 0$ from either the right- or left-hand side leads to a larger and larger slope near $\tau = 0$, which approaches infinity near $x = 0$. Thus focusing our attention, for example, on the optically thin limit we observe the following result: e/e_e at $\tau = 0^+$ is 1.75; for $\tau = 0$, $e/e_e = 1.5$ for $x = 0^+$, 1.75 for $x = 0$ and 2.0 for $x = 0^-$. This result can be easily explained on physical grounds by considering a volume element of gas adjacent to the wall. At $x = 0^+$, the element is influenced directly by the right wall only; at $x = 0^-$ the element will be influenced by the left plate only, whereas being exactly in the middle it received radiation from the right and the left plates equally.

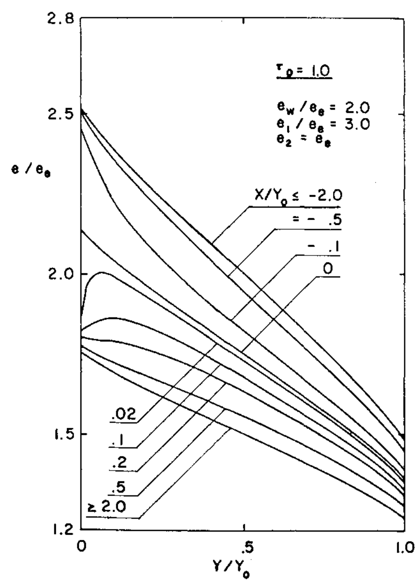

Fig. 7 Emissive power profile for $\tau_0 = 1.0$.

The heat fluxes at the lower wall $\tau = 0$ and near the upper wall $\tau = \tau_0$ are given in Fig. 8. Decreasing x decreases the net heat flux for the right wall because of the higher gaseous radiation towards the wall owing to higher temperatures of the gas near plate 1. At $x = 0$, we get a discontinuity in the net heat flux as would be expected. Because of the two-dimensional nature of this problem, the net heat flux at the upper plate is different than for the lower plate and only the integrated flux, namely the total net heat flow, is the same. Since the upper plate is at uniform temperature for this particular case there is no discontinuity at $x = 0$.

For the thin limit case the heat flux in the Y direction at any point is given by

$$q_r = e_w - (e_w - e_1)G(0,r) - e_e + (e_e - e_2)G(0,r_0) \quad X > 0 \quad (21)$$

and

$$q_r = e_1 - (e_1 - e_w)G(0,-r) - e_2 + (e_2 - e_e)G(0,-r_0) \quad X < 0 \quad (22)$$

where $G(0,r)$ is in effect the view factor and is given by Fig. 4 or an approximation thereof, using Eqs. (14) and (18).

Since there is no exact solution to this problem, it is difficult to comment on the degree of accuracy achieved by this method. It is felt, however, that one can use this method safely for small optical thicknesses and for the cases where $e_w \simeq e_1$ and $e_e \simeq e_2$. We do note that in our example for $\tau_0 = 1$ we did not restrict ourselves to the aforementioned conditions and indeed the exact numerical values obtained may be subject to error. At any rate, the results seem to be reasonable and demonstrate well the two-dimensionality character of this problem.

Concluding Remarks

The present report deals with two-dimensional radiant transfer and presents meaningful expressions for the divergence of the radiant flux [Eq. (6)] and the radiant flux perpendicular to the surface [Eq. (9)] for a plane geometry with nonisothermal walls, as described in Fig. 1.

The applicability of these expressions is demonstrated for a radiative equilibrium problem. The formulation is simple enough to yield a solution with minimum numerical efforts (compared to a rigorous analysis), and may even be sus-

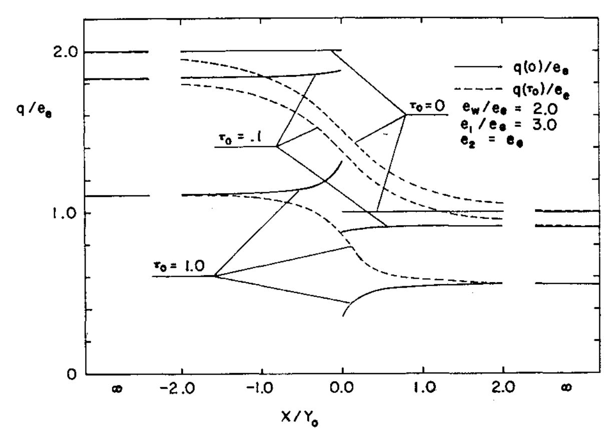

Fig. 8 Heat flux at the upper and lower plates.

ceptible to further simplifications from which a closed form solution may be achieved, as for example in the case of the optically thin limit. Though the formulation is approximate in nature, it approaches the exact solution in the optically thin limit.

It is felt that this method can be extended easily to include walls of more than two isothermal sections. In fact, for the case of radiative equilibrium where the equations are linear, a superposition technique can be used.

References

[1] Goulard, R. and Goulard, M., "One-Dimensional Energy Transfer in Radiant Media," *International Journal of Heat and Mass Transfer*, Vol. 1, 1960, p. 81.

[2] Sparrow, E. M., Usiskin, C. M., and Hubbard, H. A., "Radiation Heat Transfer in a Spherical Enclosure Containing a Participating, Heat-Generating Gas," *Journal of Heat Transfer*, Vol. 83, 1961, p. 240.

[3] Heaslet, M. A. and Warming, R. F., "Radioactive Transport and Wall Temperature Slip in an Absorbing Planar Medium," *International Journal of Heat and Mass Transfer*, Vol. 8, 1965, p. 979.

[4] Viskanta, R. and Grosh, R. J., "Heat Transfer by Simultaneous Conduction and Radiation in an Absorbing Medium," *Journal of Heat Transfer*, Vol. 84, 1962, p. 63.

[5] Greif, R., "Energy Transfer by Radiation and Conduction with Variable Gas Properties," *International Journal of Heat and Mass Transfer*, Vol. 7, 1964, p. 891.

[6] Viskanta, R., "Heat Transfer in Couette Flow of a Radiating Fluid with Viscous Dissipation," *Development in Mechanics*, edited by S. Ostrach, Pergamon Press, Oxford, 1964, p. 374.

[7] Novotny, J. L., Taitel, Y., and Hartnett, J. P., "Mass Transfer Cooling in High-Speed Couette Flow of an Absorbing Emitting Gas," *Proceedings of the 1965 Heat Transfer and Fluid Mechanics Institute*, p. 70.

[8] Sparrow, E. M. and Cess, R. D., *Radiation Heat Transfer*, Brooks/Cole, Belmont, Calif., 1966.

[9] Taitel, Y. and Hartnett, J. P., "Equilibrium Temperatures in a Boundary-Layer Flow Over a Flat Plate of Absorbing-Emitting Gas," *Journal of Heat Transfer*, Vol. 90, 1968, p. 257.

[10] Pai, S. I. and Tsao, C. K., "A Uniform Flow of a Radiating Gas over a Flat Plate," *Third International Heat Transfer Conference*, US-ASME 164, Chicago, Aug. 1966.

[11] Tsao, C. K., "A Method for Two-Dimensional Radiation Effect," *AIAA Journal*, Vol. 5, No. 1, Jan. 1967, pp. 161–163.

[12] Deissler, R. G., "Diffusion Approximation for Thermal Radiation in Gases with Jump Boundary Conditions," *Journal of Heat Transfer*, Vol. 86, 1964, p. 240.

[13] Ping, C., "Two-Dimensional Radiation Gas Flow by a Moment Method," *AIAA Journal*, Vol. 2, No. 9, Sept. 1964, pp. 1662–1664.

[14] Chang, Y. P., "A Formulation of Three-Dimensional Energy Transfer in Combined Radiation and Conduction and Methods of Solutions," Research Rept. 16, Sept. 1966, State University of New York at Buffalo.

[15] Olfe, D. B., "A Modification of the Differential Approximation for Radiative Transfer," *AIAA Journal*, Vol. 5, No. 4, April 1967, pp. 638–643.

[16] DeSoto, S. and Edwards, D. K., "Radiative Emission and Absorption in Non-Isothermal Nongray Gases in Tubes," *Proceeding of the Heat Transfer and Fluid Mechanics Institute*, Stanford University Press, June 1965.

[17] Bobco, R. P., "Directional Emissivities From a Two-Dimensional Absorbing-Scattering Medium: The Semi-Infinite Slab," Paper 67-HT-12, 1967, American Society of Mechanical Engineers.

[18] DeSoto, Simon, "Coupled Radiation Conduction and Convection in Entrance Region Flow," AIAA Paper 66-136, New York, 1966.